Induction Machines Modeling with Meshless Methods

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Abstract — Meshless Methods, also called Meshfree Methods are a class of numerical methods to solve partial differential equations. The main characteristic of these methods is that they do not need a mesh like the one used in the Finite Element Method. In this sense meshless methods are very useful for modeling moving structures, such as electric machines, without a remeshing process. In this work the Element-Free Galerkin Method is used to simulate a three phase induction motor model that includes the field circuit coupling equations and the rotor movement.

I. INTRODUCTION

Meshless methods are robust techniques used to study field problems. In these methods there is no need to explicitly establish connectivity relationships among the discretization nodes: there is no mesh, as it happens in finite element (FEM) or finite difference methods. It is only necessary a cloud of nodes that cover the problem domain [1]–[4]. This class of method is increasingly being used for doing electromagnetic field computations, especially because they can deal with changing geometry easier than other numerical methods such as the Finite Element Method. In this sense, one of the potential applications for meshless methods are electrical machines.

In our previous works we have developed some new techniques that are useful to model electrical machines using meshless methods, including the treatment of periodic boundary conditions [4] and material discontinuities [5]. In this paper we propose a new approach for the electrical machines modeling. Our technique uses the Element-Free Galerkin Method (EFGM) to solve the field-circuit coupling equations and the movement modeling.

II. ELEMENT-FREE GALERKIN METHOD

The Element-Free Galerkin Method is a meshfree method whose major features are: 1) Moving least squares (MLS) are often employed for the construction of the shape functions; 2) Galerkin weak form is employed to develop the system of equations; 3) A background mesh is required to carry out the integration used to build the system matrices [1] - [3].

In EFGM the shape functions are constructed using the MLS approximation [3], which uses a local approximation function given by:

$$u^h(\mathbf{x}) = \sum_{j=1}^{m} \mathrm{p}_j(\mathbf{x})\mathrm{a}_j(\mathbf{x}) = \underbrace{\{1\ \mathrm{x}\ \mathrm{y} \cdots \mathrm{p}_m(\mathrm{x})\}}_{\mathbf{p^T(x)}} \underbrace{\begin{Bmatrix} \mathrm{a}_1(\mathbf{x}) \\ \vdots \\ \mathrm{a}_m(\mathbf{x}) \end{Bmatrix}}_{\mathbf{a(x)}} \quad (1)$$

where $\mathbf{x}^\mathbf{T}$=[x,y] and $\mathbf{p(x)}$ is a vector of monomial basis.

The unknown parameters $\mathbf{a}(\mathbf{x})$ are determined minimizing the weighted discrete L_2 norm given by:

$$J = \sum_{I=1}^{n} w(\mathbf{x}-\mathbf{x}_I)\left[\mathbf{p}^\mathbf{T}(\mathbf{x}_I)\mathbf{a}(\mathbf{x}) - u_I\right]^2 \quad (2)$$

where w is the weight function.

Equation 2 leads to $\mathbf{a}(\mathbf{x}) = \mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{u}$ (3)

where $$\mathbf{A(x)} = \sum_{I=1}^{n} w(\mathbf{x}-\mathbf{x}_I)\mathbf{p}(\mathbf{x}_I)\mathbf{p}^\mathbf{T}(\mathbf{x}_I) \quad (4)$$

and $$\mathbf{B(x)} = \left[w(\mathbf{x}-\mathbf{x}_I)\mathbf{p}(\mathbf{x}_I)\ldots w(\mathbf{x}-\mathbf{x}_n)\mathbf{p}(\mathbf{x}_n)\right]. \quad (5)$$

By substituting (3) into (1)

$$u^h(\mathbf{x}) = \mathbf{p}^\mathbf{T}(\mathbf{x})\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{u} \quad (6)$$

or $$u^h(\mathbf{x}) = \sum_{I=1}^{n} \phi_I(\mathbf{x})u_I \quad (7)$$

where $$\phi_I(\mathbf{x}) = \mathbf{p}^\mathbf{T}\mathbf{A}^{-1}\mathbf{B}_I \quad (8)$$

is the MLS shape function.

III. THE ELECTRICAL MACHINE MODEL

A three-phase, 4 poles, 50Hz, 2 HP squirrel-cage induction motor is modeled. Due to symmetry considerations and using anti-periodic boundary conditions, only ¼ of it needs to be modeled. The original motor has a total of 36 stator slots with 44 Amp*Turns. There are 28 slots on the rotor filled with aluminum (σ = 34.45 MS/m) with 100mm depth [7]. Figure 1 shows the motor geometry used in this work.

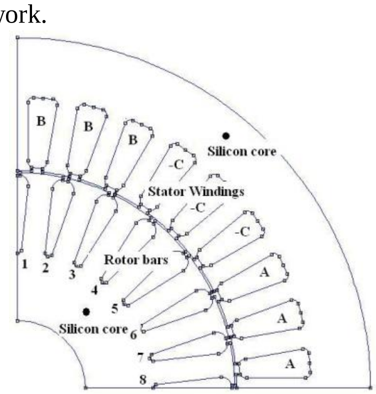

Fig. 1. Motor geometry

The field-circuit equations for this machine are expressed as [6]:

$$\nabla \cdot \nu\nabla A - \sigma\frac{\partial A}{\partial t} + J_s = 0 \quad (9)$$

$$\nabla \cdot \nu \nabla A - \sigma \frac{\partial A}{\partial t} + \sigma \frac{U_t}{l} = 0 \tag{10}$$

and

$$U_t = R_t I_t + R_t \int_{S_t} \sigma \frac{\partial A}{\partial t} ds. \tag{11}$$

A is the magnetic vector potential, ν is the magnetic reluctivity, σ is the conductivity, J_s is the current density source represented in figure 1 by the phases A, B and C. S_t is the section of each rotor conductor, l is its length, U_t is the voltage on the bar terminals and I_t is the current on it.

The Element-Free Galerkin method is applied to equations (9), (10) and (11), with shape functions given by (8). After this and considering all rotor bars of the study domain, a set of matrix equations is obtained as follows:

$$\mathbf{KA} + \mathbf{N}\frac{d}{dt}\mathbf{A} - \mathbf{PU_t} - \mathbf{J} = 0 \tag{12}$$

$$\mathbf{Q}\frac{d}{dt}\mathbf{A} + \mathbf{C_3 U_t} + \mathbf{RI_t} = 0 \tag{13}$$

$$\mathbf{C_1^T C_1 U_t} + \mathbf{C_2 I_t} = 0 \tag{14}$$

where **K** is the stiffness matrix, **N** is the matrix related to shape functions, $\mathbf{C_1}$ $\mathbf{C_2}$ and $\mathbf{C_3}$ are auxiliary matrix obtained by Kirchhoff's laws and **P** and **Q** are matrix related to rotor bars [6].

The movement modeling consists basically in rotor nodes translation, with their new positions evaluated each time step using the rotor speed. Another important aspect that should be observed is the periodic boundaries that are constantly changing. For this, a special treatment is necessary and will be discussed in detail in the extended version of this paper.

IV. PRELIMINARY RESULTS

The results were obtained using the model presented in the previous section. Initially, the machine is locked. The induced current in bar 6, evaluated by EFGM, is shown in figure 2 (see figure 1 to identify bar numbers). For validation, the results obtained by FEMM, a finite element software that deal with field-circuit coupling using a classical approach [7], are also presented.

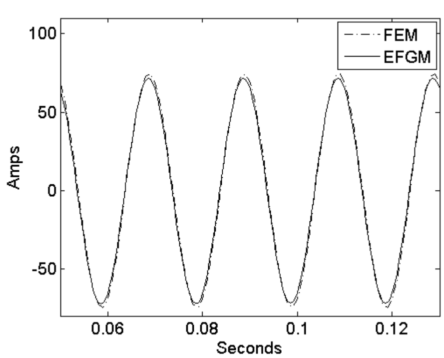

Fig. 2. Current in rotor bar n^0 6 - FEM/EFGM.

Figures 3 and 4 show the machine flux distribution in two different time moments. The machine is running with 10% slip. The simulations use anti-periodic boundary conditions.

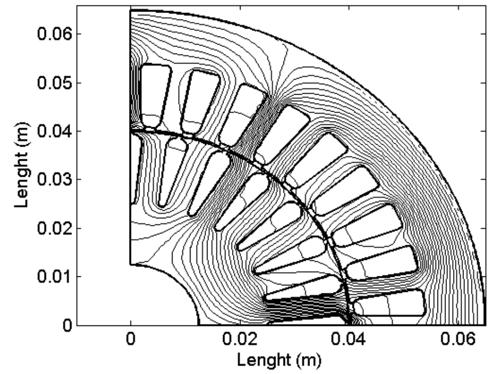

Fig. 3. Resulting magnetic flux distribution at the initial moment.

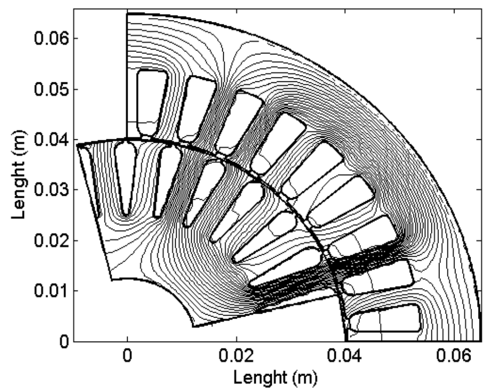

Fig. 4. Resulting magnetic flux distribution after 1,65ms.

V. CONCLUSION

In this paper, a new approach for induction machines modeling was proposed using the Element-Free Galerkin Method. The movement implementation, one difficult task to other methods, was done considering essentially the rotor nodes translation. In this sense meshless methods show their contribution in solving this important electromagnetic problem.

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